AP Calculus AB Name:______________________

Review 2, No Calculator

Complete all the following on notebook paper. *Solution follow this page

A 1.

The point on the curve $x^2+2y=0$ that is nearest the point $\left(0,-\frac{1}{2}\right)$ occurs where y is

(A) $\frac{1}{2}$ (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) none of the above

A 2.

If $f(x)=\frac{4}{x-1}$ and $g(x)=2x$, then the solution set of $f(g(x))=g(f(x))$ is

(A) $\left\{\frac{1}{3}\right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1,2\}$ (E) $\left\{\frac{1}{3},2\right\}$

C 3.

The region bounded by the x-axis and the part of the graph of $y=\cos x$ between $x=-\frac{\pi}{2}$ and $x=\frac{\pi}{2}$ is separated into two regions by the line $x=k$. If the area of the region for $-\frac{\pi}{2}\le x\le k$ is three times the area of the region for $k\le x\le\frac{\pi}{2}$, then $k=$

(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

C 4.

If the function f is defined by $f(x)=x^5-1$, then f^{-1}, the inverse function of f, is defined by $f^{-1}(x)=$

(A) $\frac{1}{\sqrt[5]{x}+1}$ (B) $\frac{1}{\sqrt[5]{x+1}}$ (C) $\sqrt[5]{x-1}$

(D) $\sqrt[5]{x}-1$ (E) $\sqrt[5]{x+1}$

A 5.

If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real x, then the graph of $y = f(x)$ and the graph of $y = g(x)$

(A) intersect exactly once.

(B) intersect no more than once.

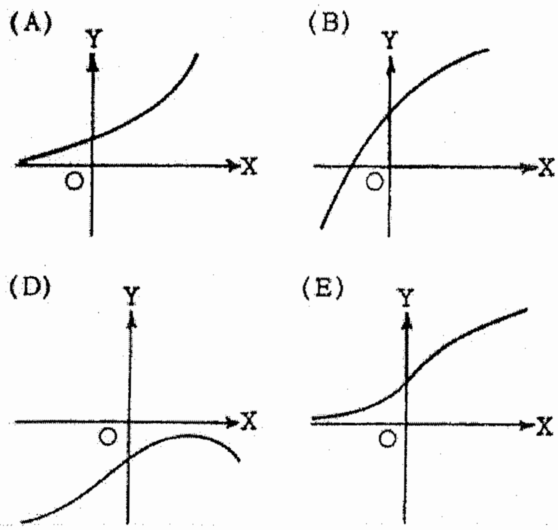
(C) do not intersect.

(D) could intersect more than once.

(E) have a common tangent at each point of intersection.

B 6.

If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



B 7.

The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) $(0,0)$ only (B) $(3,162)$ only (C) $(4,256)$ only

(D) $(0,0)$ and $(3,162)$ (E) $(0,0)$ and $(4,256)$

E 8.

If $f(x) = 2 + |x-3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

(A) -1 (B) 0 (C) 1 (D) 2 (E) nonexistent

C 9.

A point moves on the x-axis in such a way that its velocity at time t $(t>0)$ is given by $v=\dfrac{\ln t}{t}$. At what value of t does v attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

A 10.

An equation for a tangent to the graph of $y=\arcsin\dfrac{x}{2}$ at the origin is

(A) $x-2y=0$ (B) $x-y=0$ (C) $x=0$ (D) $y=0$ (E) $\pi x-2y=0$

11. 2000—AB6

Consider the differential equation $\dfrac{dy}{dx}=\dfrac{3x^2}{e^{2y}}$.

(a) Find a solution $y=f(x)$ to the differential equation satisfying $f(0)=\dfrac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

12. 2001—AB4

Let h be a function defined for all $x \neq 0$ such that $h(4)=-3$ and the derivative of h is given by $h'(x)=\dfrac{x^2-2}{x}$ for all $x \neq 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x=4$.

(d) Does the line tangent to the graph of h at $x=4$ lie above or below the graph of h for $x>4$? Why?

Review 2

1. $x^2+2y=0$

$x^2=-2y$

$d=\sqrt{(x-x)^2+(y-y)^2}$

$d=\sqrt{(x-0)^2+(y+1/2)^2}$

$d=\sqrt{x^2+y^2+y+\frac{1}{4}}$

$d=\sqrt{-2y+y^2+y+\frac{1}{4}}$

$d=\sqrt{y^2-y+\frac{1}{4}}$

$d'=\dfrac{2y-1}{2\sqrt{y^2-y+\frac{1}{4}}}$

$d'=0 \quad y=\frac{1}{2} \qquad d' \text{ dne } \quad y=\frac{1}{2}$

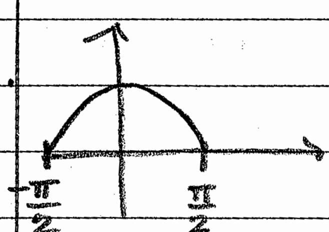
d' ← − | + → at $1/2$ $\boxed{A}$

2. $f(2x)=\dfrac{4}{2x-1}$ $\qquad g\left(\dfrac{4}{x-1}\right)=\dfrac{8}{x-1}$

$\dfrac{4}{2x-1}=\dfrac{8}{x-1} \longrightarrow 4x-4=16x-8$

$4=12x$

$x=1/3$ $\boxed{A}$

3. 

$\int_{-\frac{\pi}{2}}^{k} \cos x \, dx = 3\int_{k}^{\frac{\pi}{2}} \cos x \, dx$

$\sin x \Big|_{-\frac{\pi}{2}}^{k} = 3\sin x \Big|_{k}^{\frac{\pi}{2}}$

$\sin k - \sin\frac{-\pi}{2} = 3\sin\frac{\pi}{2} - 3\sin k$

$\sin k + 1 = 3 - 3\sin k$

$4\sin k = 2$

$\sin k = \frac{1}{2}$

$k=\frac{\pi}{6}$ $\boxed{C}$

4. $f(x) = x^5 - 1$

$x = y^5 - 1$

$y^5 = x + 1$

$y = \sqrt[5]{x+1}$ $\boxed{C}$

5. $\boxed{A}$

6. inc and cd $\boxed{B}$

7. $y = 5x^4 - x^5$

$y' = 20x^3 - 5x^4$

$y'' = 60x^2 - 20x^3 = 20x^2(3-x)$

$y'' = 0 \quad x = 0, 3 \qquad y'' \; \overset{+}{\longleftarrow}\big|_0 \overset{+}{\longrightarrow}\big|_3 \overset{-}{\longrightarrow}$ $\boxed{B}$

8. $\boxed{E}$ graph → ~~there is a corner at x=3~~

9. $v = \frac{\ln t}{t}$

$a = \frac{t\left(\frac{1}{t}\right) - \ln t}{t^2} = \frac{1 - \ln t}{t^2}$ $\qquad a = 0 \rightarrow 1 - \ln t = 0$

$\ln t = 1$

$t = e$

a dne $t = 0$

$a \quad \longleftarrow \big|_0 \overset{+}{\longrightarrow} \big|_e \overset{-}{\longrightarrow}$ $\boxed{C}$

10. $y' = \frac{\frac{1}{2}}{\sqrt{1 - \frac{x^2}{4}}}$ $\qquad y'(0) = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x - 0)$ $\boxed{A}$

Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

(a) $e^{2y}\,dy = 3x^2\,dx$

$$\frac{1}{2}e^{2y} = x^3 + C_1$$

$$e^{2y} = 2x^3 + C$$

$$y = \frac{1}{2}\ln\left(2x^3 + C\right)$$

$$\frac{1}{2} = \frac{1}{2}\ln(0 + C);\quad C = e$$

$$y = \frac{1}{2}\ln(2x^3 + e)$$

6 $\begin{cases} 1: \text{ separates variables} \\ 1: \text{ antiderivative of } dy \text{ term} \\ 1: \text{ antiderivative of } dx \text{ term} \\ 1: \text{ constant of integration} \\ 1: \text{ uses initial condition } f(0) = \frac{1}{2} \\ 1: \text{ solves for } y \\ \text{Note: } 0/1 \text{ if } y \text{ is not a logarithmic function of } x \end{cases}$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(b) Domain: $2x^3 + e > 0$

$$x^3 > -\frac{1}{2}e$$

$$x > \left(-\frac{1}{2}e\right)^{1/3} = -\left(\frac{1}{2}e\right)^{1/3}$$

Range: $-\infty < y < \infty$

3 $\begin{cases} 1: \ 2x^3 + e > 0 \\ 1: \text{ domain} \\ \quad \text{Note: } 0/1 \text{ if } 0 \text{ is not in the domain} \\ 1: \text{ range} \end{cases}$

Note: 0/3 if y is not a logarithmic function of x

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Question 4

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

(a) $h'(x) = 0$ at $x = \pm\sqrt{2}$

$h'(x)$	$-$	0	$+$	und	$-$	0	$+$
x		$-\sqrt{2}$		0		$\sqrt{2}$	

Local minima at $x = -\sqrt{2}$ and at $x = \sqrt{2}$

$4: \begin{cases} 1: x = \pm\sqrt{2} \\ 1: \text{analysis} \\ 2: \text{conclusions} \\ \quad < -1 > \text{ not dealing with discontinuity at } 0 \end{cases}$

(b) $h''(x) = 1 + \dfrac{2}{x^2} > 0$ for all $x \neq 0$. Therefore, the graph of h is concave up for all $x \neq 0$.

$3: \begin{cases} 1: h''(x) \\ 1: h''(x) > 0 \\ 1: \text{answer} \end{cases}$

(c) $h'(4) = \dfrac{16 - 2}{4} = \dfrac{7}{2}$

$y + 3 = \dfrac{7}{2}(x - 4)$

1 : tangent line equation

(d) The tangent line is below the graph because the graph of h is concave up for $x > 4$.

1 : answer with reason